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Thermal shock problem for a fiber-reinforced anisotropic half-space placed in a magnetic field via G-N model

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ABSTRACT

The generalized magneto-thermoelasticity theory, based on Green–Naghdi model, is used to study the thermal shock problem of a fiber-reinforced anisotropic half-space. The solid half-space deforms because of thermal shock, and due to the application of the magnetic field. Maxwell's equations are formulated and the generalized coupled governing equations are derived. Finite element method with the Laplace transform technique is used to obtain the general solution. The distributions of the considered physical variables are represented graphically. The influence of the magnetic field is discussed. Some particular cases of special interest due to the application of two types of Green and Naghdi's theory have been deduced.

1. Introduction

Thermal shock is usually modeled using analytical solution for the temperature profiles [1]. The thermal boundary conditions for this solution may be instantaneous change in surface temperature, constant convective heat transfer coefficient, or constant heating (cooling) rate. Many earlier studies on thermal problems considered thermal shock for composite structures or fiber-reinforced half-spaces. For example, Lessen [2] formulated the problem of initial thermal shock and found that the solution is similar to that of the Fourier equation with a modified diffusivity. Dolotov and Kill [3] obtained the small-time asymptotic form of the exact solution of the axially symmetric problem of the thermal shock at the boundary of an elastic half-space. Mukherjee and Sinha [4] examined the coupled dynamic thermoelastic response of a fibrous composite plate exposed to a thermal shock by using the finite element method. Tianhu et al. [5] used the theory of generalized thermoelasticity, based on Lord-Shulman theory with one relaxation time [6] to study the electromagneto-thermoelastic interactions in a semi-infinite perfectly conducting solid. This solid is subjected to a thermal shock on its surface when it and its adjoining vacuum are subjected to a uniform axial magnetic field. Othman [7] considered a 2-D coupled problem in electromagneto-thermoelasticity for a thermally and electrically conducting half-space solid whose surface is subjected to a thermal shock. He used Lord-Shulman generalized thermoelasticity theory with one relaxation time [6]. Ezzat and Youssef [8] established a 3-D model of the generalized thermoelasticity with one relaxation time for a specific problem of a half space subjected to thermal shock and traction free surface. Sarkar and Lahiri [9] considered a 3-D problem for a homogeneous, isotropic and thermoelastic half-space. This half-space is subjected to a time-dependent heat source on the traction-free boundary of

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the space. They used the generalized thermoelasticity theory based on Green–Naghdi model II [10] (thermoelasticity without energy dissipation).

It is known from now that the classical coupled thermoelasticity theory leads to a parabolic type heat conduction equation, called the diffusion equation. This theory was proposed by Biot [11] with the introduction of the strain-rate term in the Fourier heat conduction equation. It predicts finite propagation speed for elastic wave (physically unrealistic) but an infinite speed for thermal disturbance. Different generalized thermoelasticity theories are presented to overcome such an absurdity (see, e.g., Lord–Shulman [6] and Green–Lindsay [12] theories) and advocating the existence of finite thermal wave speed in solids. Many researchers developed these theories by introducing one or two relaxation times in the thermoelastic process, either by modifying Fourier's heat conduction equation or by correcting the energy equation and Neumann–Duhamel relation. The Lord–Shulman theory is based on the modified Fourier's law of heat conduction, and admits one relaxation time. However, the Green–Lindsay theory modifies both the energy equation and the Neumann–Duhamel relation, and allows two relaxation times. Lastly, Green and Naghdi [10,13] provide sufficient basic modifications in the constitutive equations that permit treatment of a much wider class of heat flow problems. The development is quite general, and the characterization of the material response for the thermal phenomena is based on three types of the constitutive function (see Green and Naghdi [14–16]). The natures of these types of constitutive equations are such that when the respective theories are linearized, type I is the same as the classical heat equation (based on Fourier's law) whereas the linearized versions of types II and III permit propagation of thermal waves at finite speed. The entropy flux vector in types II and III (i.e. thermoelasticity without energy dissipation and thermoelasticity with energy dissipation) models are determined in terms of the potential that also determines stresses.

The influence of the primary magnetic field to the theory of thermoelasticity has received attention of many researchers due to its widely applications. Sherief and Ezzat [17] considered the 1-D problem of generalized thermoelastic electrically conducting half-space permeated by a primary uniform magnetic field when the bounding plane is suddenly heated to a constant temperature. Baksi et al. [18] examined magneto-thermoelastic problems with thermal relaxation and heat sources in a 3-D infinite rotating elastic medium. Lotfy [19] and Othman and Lotfy [20] studied the influences of a magnetic field and rotation on a two-dimensional problem of fiber-reinforced thermoelasticity based on the coupled theory [11], Lord–Shulman theory [6], and Green–Lindsay theory [12].

In the present work, the Green–Naghdi theory is applied to study the influence of the magnetic field for the thermal shock problem of a fiber-reinforced anisotropic half-space. The problem has been solved numerically using a finite element method. The distributions of the temperature, displacements and stresses are represented graphically. Numerical results for the field quantities are given and illustrated in the presence and absence of the magnetic field using both types of Green–Naghdi theory.

2. Formulation of the problem

Let us consider the problem of a perfectly conducting thermoelastic half-space ($x \geqslant 0$). A magnetic field with primary constant intensity $\vec{H} = (0, 0, H_0)$ is acting parallel to the bounding plane (taken as the direction of the z-axis). The surface of the half-space is subjected to a thermal shock which is a function of y and t. Thus, all the quantities considered will be functions of the time variable t, and of the coordinates x and y. Due to the application of the initial magnetic field $\vec{H}$, there results an induced magnetic field h and an induced electric field $\vec{E}$. Let us begin this consideration with the linearized equations of electro-dynamics of slowly moving medium [7]

$$\vec{J} = \mathrm{curl}\vec{h} - \varepsilon_0\dot{\vec{E}}, \tag{1}$$

$$\mathrm{curl}\ \vec{E} = -\mu_0\dot{\vec{h}} \tag{2}$$

$$\vec{E} = -\mu_0(\dot{\vec{u}} \times \vec{H}), \tag{3}$$

$$\mathrm{div}\ \vec{h} = 0, \tag{4}$$

where μ_0 is the magnetic permeability; ε_0 is the electric permeability, $\dot{\vec{u}}$ is the particle velocity of the medium, $\vec{h}$ is the induced magnetic field vector, $\vec{E}$ is the induced electric field vector and $\vec{J}$ is the current density vector. These equations are supplemented by the displacement equations of the theory of elasticity, taking into consideration the Lorentz force F_i to give

$$\sigma_{ij,j} + F_i = \rho\ddot{u}_i, \tag{5}$$

$$F_i = \mu_0(\vec{J} \times \vec{H})_i, \tag{6}$$

where σ_{ij} is the stress tensor, u_i are the displacement components and ρ is the mass density. The comma notation is used for spatial derivatives and superimposed dot represents time differentiation. The constitutive equation for a fiber-reinforced linearly thermoelastic anisotropic medium whose preferred direction is that of a unit vector $\vec{a}$ is

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j - \beta_{ij}(T - T_0)\delta_{ij},\quad i,j,k,m = 1,2,3. \tag{7}$$

where T is the temperature change of a material particle, T_0 is the reference uniform temperature of the body, β_{ij} is the thermal elastic coupling tensor, δ_{ij} is the Kronecker delta, λ and μ_T are elastic parameters, α, β and $(\mu_L - \mu_T)$ are reinforced anisotropic elastic parameters and $\vec{a} \equiv (a_1, a_2, a_3), a_1^2 + a_2^2 + a_3^2 = 1$. The heat conduction equation

$$K^* T_{,ii} + K_{ij}\dot{T}_{,ij} = \rho c_e \ddot{T} + T_0 \ddot{u}_{i,j}, \tag{8}$$

where c_e is the specific heat at constant strain, K_{ij} are the thermal conductivity components and K^* is the material characteristic of the theory. For the problem of a thermoelastic half-space ($x \geqslant 0$) in the context of Green-Naghdi's (GNIII and GNII) generalized thermoelasticity theory (with and without energy dissipation), all the considered functions will be depend on the time t and the coordinates x and y. Thus, the displacement components u_i will be

$$u_x = u(x,y,t), \quad u_y = v(x,y,t), \quad u_z = 0. \tag{9}$$

Let us choose the fiber-direction as $\vec{a} \equiv (1,0,0)$ so that the preferred direction is the x-axis and Eqs. (5)–(7) are simplified as

$$\sigma_{xx} = \left(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta\right)\frac{\partial u}{\partial x} + (\lambda + \alpha)\frac{\partial v}{\partial y} - \beta_{11}(T - T_0), \tag{10}$$

$$\sigma_{yy} = (\lambda + \alpha)\frac{\partial u}{\partial x} + (\lambda + 2\mu_T)\frac{\partial v}{\partial y} - \beta_{22}(T - T_0), \tag{11}$$

$$\sigma_{xy} = \mu_L\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \tag{12}$$

$$F_x = \mu_0 H_0^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} - \varepsilon_0\mu_0\frac{\partial^2 u}{\partial t^2}\right), \tag{13}$$

$$F_y = \mu_0 H_0^2\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} - \varepsilon_0\mu_0\frac{\partial^2 v}{\partial t^2}\right), \tag{14}$$

$$(A_{11} + \rho R_H^2)\frac{\partial^2 u}{\partial x^2} + (A_{12} + \rho R_H^2)\frac{\partial^2 v}{\partial x \partial y} + A_{13}\frac{\partial^2 u}{\partial y^2} - \beta_{11}\frac{\partial T}{\partial x} = \rho\left(1 + \frac{R_H^2}{c^2}\right)\frac{\partial^2 u}{\partial t^2}, \tag{15}$$

$$(A_{22} + \rho R_H^2)\frac{\partial^2 v}{\partial y^2} + (A_{12} + \rho R_H^2)\frac{\partial^2 u}{\partial x \partial y} + A_{13}\frac{\partial^2 v}{\partial x^2} - \beta_{22}\frac{\partial T}{\partial y} = \rho\left(1 + \frac{R_H^2}{c^2}\right)\frac{\partial^2 v}{\partial t^2}, \tag{16}$$

$$\left(K^* + K_{11}\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x^2} + \left(K^* + K_{22}\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial y^2} = \rho c_e\frac{\partial^2 T}{\partial t^2} + T_0\frac{\partial^2}{\partial t^2}\left(\beta_{11}\frac{\partial u}{\partial x} + \beta_{22}\frac{\partial v}{\partial y}\right), \tag{17}$$

where

$$\begin{aligned} &A_{11} = \lambda + 2(\alpha + \mu_T) + 4(\mu_L - \mu_T) + \beta, \quad R_H^2 = \frac{\mu_0 H_0^2}{\rho}, \\ &A_{12} = \alpha + \lambda + \mu_L, \quad A_{13} = \mu_L, \quad A_{22} = \lambda + 2\mu_T, \quad c^2 = \frac{1}{\varepsilon_0\mu_0}, \\ &\beta_{11} = (2\lambda + 3\alpha + 4\mu_L - 2\mu_T + \beta)\alpha_{11}, \quad \beta_{22} = (2\lambda + \alpha)\alpha_{11} + (\lambda + 2\mu_T)\alpha_{22}, \end{aligned} \tag{18}$$

in which α_{11} and α_{22} are coefficients of linear thermal expansion. In what follows, it is convenient now to introduce the following dimensionless variables:

$$\begin{aligned} &(x', y', u', v') = c_1\chi(x, y, u, v), \quad t' = c_1^2\chi t, \quad T' = \frac{\beta_{11}(T - T_0)}{\rho c_1^2}, \quad \chi = \frac{\rho c_e}{K_{11}}, \\ &(\sigma'_{xx}, \sigma'_{xy}, \sigma'_{yy}) = \frac{1}{\rho c_1^2}(\sigma_{xx}, \sigma_{xy}, \sigma_{yy}), \quad h' = \frac{h}{H_0}, \quad c_1^2 = \frac{A_{11}}{\rho}. \end{aligned} \tag{19}$$

In terms of the non-dimensional quantities defined in Eqs. (19), the above governing equations reduce to (dropping the dashed for convenience)